

Domain Formation in $\nu = 2/3$ Fractional Quantum Hall Systems

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We study the domain formation in the $\nu = 2/3$ fractional quantum Hall systems basing on the density matrix renormalization group (DMRG) analysis. The ground-state energy and the pair correlation functions are calculated for various spin polarizations. The results confirm the domain formation in partially spin polarized states, but the presence of the domain wall increases the energy of partially spin polarized states and the ground state is either spin unpolarized state or fully spin polarized state depending on the Zeeman energy. We expect coupling with external degrees of freedom such as nuclear spins is important to reduce the energy of partially spin polarized state.

KEYWORDS: fractional quantum hall effect, spin transition, domain structure, density matrix, renormalization group

The spin degrees of freedom of electrons in quantum Hall systems play an essential role in the ground state and low-energy excitations.¹⁾ In two-dimensional systems, perpendicular magnetic field completely quenches the kinetic energy of the electrons, and the exchange Coulomb interaction easily aligns the spins of electrons. The ferromagnetic ground state at the filling $\nu = 1/q$ (q odd) is thus realized even in the absence of the Zeeman splitting.¹⁾ At the filling $\nu = 2/3$ and $2/5$, however, the ferromagnetic and paramagnetic ground states compete with each other, and the spin transition between the two states is induced by the Zeeman splitting $\Delta_z = g\mu_B B$.²⁾ Such an interesting spin transition in fractional quantum Hall systems has been naively explained by composite fermion theory.³⁾ In this theory, the $\nu = p/(2p \pm 1)$ fractional quantum Hall effect (FQHE) state is mapped on to the $\nu' = p$ integer QHE state of composite fermions, that means the spin transitions at $\nu = 2/3$ and $2/5$ ⁴⁾ correspond to the spin transition at $\nu = 2$, where the upper minority spin state in the lowest Landau level (LL) and the lower majority spin state in the second lowest LL cross when the Zeeman splitting coincides with the effective LL separation.

A large number of experimental and theoretical studies have been made on this intriguing transition. Magnetotransport measurements^{5–14)} clarified that there exists a clear transition between the unpolarized state ($P = 0$) to the fully polarized state ($P = 1$) at $\nu = 2/3$ induced by tilting the field or tuning carrier density within a fixed LL filling factor. Significantly an optical experiment⁸⁾ confirmed the transition from $P = 0$ to $P = 1$, but also revealed a stable

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half polarized state around $P = 1/2$ that has been the subject of extensive theoretical studies recently.^{15–18)}

Proposed half polarized states are a $L = 1$ exciton condensate,¹⁵⁾ a crystal state¹⁶⁾ of composite fermions,³⁾ spin paired state,¹⁷⁾ and antiferromagnetic spin liquid¹⁸⁾ that have been claimed to be stabilized between $P = 0$ and $P = 1$. Nonetheless, there is no clear theoretical consensus for this transition. Sources of these discrepancies might be due to the difficulty of theoretical studies in this system; (i) A number of states possibly compete in energy, (ii) It needs a large enough system to see non-uniform structures of the partially polarized states argued above. Thus a large scale numerical study is desired to clarify the nature of the partially polarized many-body states.

In this Letter, we use the density matrix renormalization group (DMRG) method,^{19–24)} and study the spin transition and the domain formation in the $\nu = 2/3$ fractional quantum Hall systems. It is found that the spin unpolarized and fully spin polarized domains are spontaneously formed in partially spin polarized state $0 < P < 1$. However, the existence of the domain walls between the two domains rise the energy of partially spin polarized states, and the energy of the states with domains is higher than that of the ground states of $P = 0$ or $P = 1$. Thus the transition between $P = 0$ and $P = 1$ is first order, and there is no stable state between them. Our results are in contrast to the conclusions of stable half polarized states in recent theoretical studies.^{15–18)}

We assume the low-field regime where the Zeeman splitting is smaller than or comparable to the Coulomb energy e^2/l while the LL separation is still large, so that only the lowest LL is occupied by electrons. Here l is the magnetic length. The Hamiltonian of this system is written as

$$H = \sum_{i < j, \mathbf{q}} V(q) e^{-q^2 l^2 / 2} e^{i\mathbf{q} \cdot (\mathbf{R}_i - \mathbf{R}_j)} - \Delta_z \sum_i S_{i,z}, \quad (1)$$

where \mathbf{R}_i is the two-dimensional guiding center coordinates of the i th electron. The guiding center coordinates satisfy the commutation relation, $[R_i^x, R_j^y] = il^2 \delta_{i,j}$. $V(q) = 2\pi e^2/q$ is the Fourier component of the Coulomb potential. We consider uniform positive background charge to cancel the component at $q = 0$.

We calculate the ground-state wave function using the DMRG method,¹⁹⁾ which is a real space renormalization group method combined with the exact diagonalization method. The DMRG method provides the low-energy eigenvalues and corresponding eigenvectors of the Hamiltonian within a restricted number of basis states. The accuracy of the results is systematically controlled by the truncation error, which is smaller than 10^{-4} in the present calculation. We investigate systems of various sizes with up to 26 electrons in the unit cell $L_x \times L_y$ keeping 500 basis in each block.^{20–24)}

Figure 1 shows the lowest energy of the states with various polarization P as a function

of the Zeeman splitting, $\Delta_z = g\mu B$. We chose positive Δ_z in eq. (1) that means up spins are majority. In the absence of the Zeeman splitting, the unpolarized state ($P = 0$) is the lowest. The energy of polarized state ($P > 0$) monotonically increases as P increases. With the increase in Zeeman splitting Δ_z , however, the energy of polarized state decreases and in the limit of large Zeeman splitting, the fully polarized state ($P = 1$) becomes the lowest. Figure 1 shows that the transition from the unpolarized state to the fully polarized state occurs at $B \simeq 6[\text{T}]$ which is roughly consistent to the earlier work done in a spherical geometry.²⁾ Strictly speaking, in the earlier work on a sphere, a finite gapless regime has been seen between

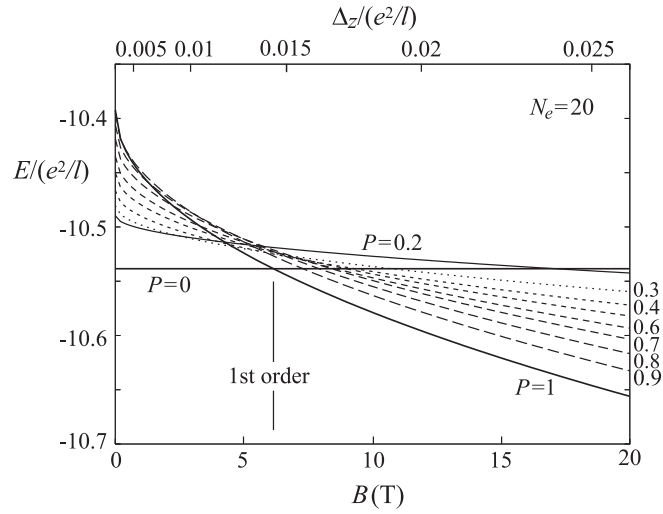


Fig. 1. Lowest energies for fixed polarization ratio P as a function of magnetic field B at filling factor $\nu = 2/3$ in units of e^2/l . The total number of electron is 20. The aspect ratio is fixed at 2.0. The g -factor is 0.44.

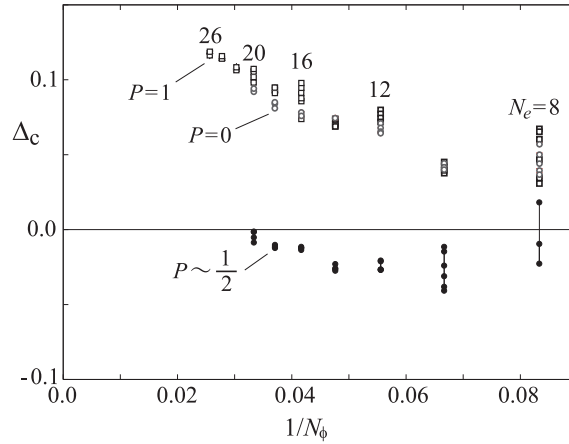


Fig. 2. Charge gap of $\nu = 2/3$ spin polarized states (\square), unpolarized states (\circ), and partially polarized states (\bullet) for various N_e and aspect ratios L_x/L_y . Δ_c is in units of e^2/l .

the $P = 0$ FQHE state and $P = 1$ FQHE state. We believe such a gapless state stems from the fact that two FQHE states with $P = 0$ and 1 realize at different ratios of the total magnetic flux N_ϕ and the number of electrons N_e . Indeed, in the present calculation on a torus, in the whole range of the Zeeman splitting, all partially polarized states ($0 < P < 1$) are higher in energy than the ground states ($P = 0$ or 1). This feature is independent on the size of the system and the aspect ratio L_x/L_y . This result indicates that phase separations of $P = 0$ domains and $P = 1$ domains might occur in partially polarized states.

The unpolarized state of $P = 0$ and the fully polarized state of $P = 1$ are both quantum Hall states with finite charge excitation gap defined by

$$\Delta_c(P) = E(N_\phi + 1, P) + E(N_\phi - 1, P) - 2E(N_\phi, P), \quad (2)$$

where N_ϕ is the number of one-particle states in the lowest Landau level. The filling factor ν , which is fixed $2/3$, is then given by N_e/N_ϕ . The charge gap Δ_c for various N_ϕ and aspect ratios of the unit cell is presented in Fig. 2. This figure shows that although Δ_c for states with $P = 0$ and 1 is finite, it seems to vanish for partially polarized state $P \sim 1/2$ in the limit of $N \rightarrow \infty$. This result clearly indicates that partially polarized state with $P \sim 1/2$ is a compressible state in contrast to the incompressible states at $P = 0$ and 1.

To study the spin structure in the partially polarized states, we calculated the pair-correlation function defined by

$$g_{\sigma\sigma}(\mathbf{r}) = \frac{L_x L_y}{N_\sigma(N_\sigma - 1)} \langle \Psi | \sum_{nm} \delta(\mathbf{r} + \mathbf{R}_{\sigma,n} - \mathbf{R}_{\sigma,m}) | \Psi \rangle, \quad (3)$$

where $\sigma = \pm 1/2$ is the spin index. The spin structure in partially spin polarized state is clearly shown in the pair correlation function between minority spins. Namely, if unpolarized domains are formed in the partially polarized states, then electrons with minority spin are

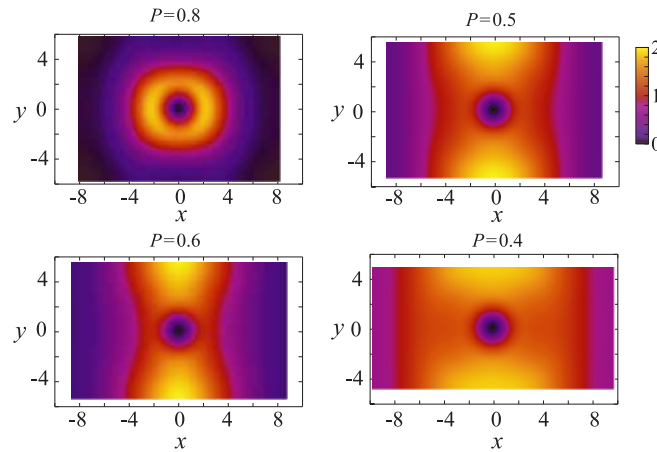


Fig. 3. Pair correlation functions for minority spins for several polarization ratios (a) $P = 0.8$, (b) $P = 0.6$, (c) $P = 0.5$, and (d) $P = 0.4$.

concentrated in the unpolarized domains. This concentration of the minority spin is actually shown in Fig. 3, which shows $g_{\downarrow\downarrow}(x, y)$ for partially polarized states at (a) $P = 0.8$, (b) $P = 0.6$, (c) $P = 0.5$, and (d) $P = 0.4$. When P is close to 1, for example $P = 0.8$ shown in Fig. 3(a), a pair of minority spins is found only near the origin. As the polarization ratio P decreases, minority spins are concentrated around the origin, and two domain walls along the y -direction is formed. These domain walls move along x -direction and the domain with minority spins finally dominates entire system in the limit of $P = 0$. This change in the size of the domain is consistent with the expectation that the domain in Fig. 3 corresponds to the unpolarized spin singlet domain where the density of up-spin electrons and the down-spin electrons are the same.

To confirm the formation of the unpolarized spin domain, we next consider the local electron density of up-spin electrons $\nu_{\uparrow}(x)$ and down-spin electrons $\nu_{\downarrow}(x)$. Figure 4 shows $\nu_{\uparrow}(x)$ and $\nu_{\downarrow}(x)$ for partially polarized states with $P = 0.2, 0.4, 0.6$ and 0.7 . Here $\nu_{\uparrow}(x)$ and $\nu_{\downarrow}(x)$ are scaled to be the local filling factor of the lowest LL. Thus, the total local electron density $\nu_{\uparrow}(x) + \nu_{\downarrow}(x)$ is almost $2/3$ everywhere. In this figure two domains are clearly seen;²⁵⁾ the unpolarized spin domain around $L_x/2$, where both ν_{\uparrow} and ν_{\downarrow} are close to $1/3$, and the fully polarized spin domain around $x \sim 0$ or equivalently $x \sim L_x$, where ν_{\uparrow} is almost $2/3$ while ν_{\downarrow} is close to 0. These results confirm the formation of the unpolarized and polarized spin domains as expected from the pair correlation functions shown in Fig. 3.

The polarized and unpolarized spin domains are separated by the domain walls whose width is about $4l$. This means the domains are realized only for systems whose size of the unit cell $L_x, (L_y)$ is larger than twice the width of domain wall; $L_x, (L_y) > 8l$. Indeed, exact diagonalization studies up to $N_e = 8$ electrons have never found the domain structure at

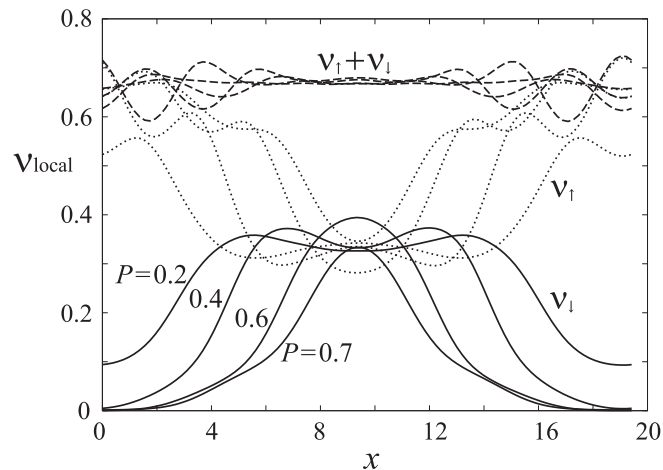


Fig. 4. Local densities of up spin, and down spin electrons for various polarization ratios P . The number of electrons is 20.

$\nu = 2/3$.¹⁸⁾ We have found the domain structure only for large systems with $N_e > 12$.

We studied the energy and the spin structure of the many-body ground-states at the filling factor $\nu = 2/3$ using the DMRG method. The obtained ground state energy for various polarization P shows that the ground state evolves discontinuously from the unpolarized $P = 0$ state to the fully polarized $P = 1$ state as the Zeeman splitting increases. In partially polarized states $0 < P < 1$, the electronic system separates spontaneously into two domains; the $P = 0$ domain and the $P = 1$ domain. The two domains are separated by the domain wall of width $4l$. Since the energy of the domain wall is positive, the partially polarized states always has higher energy than that of $P = 1$ or $P = 0$ states. We think this is the reason of the direct first order transition from $P = 0$ to $P = 1$ state in the ground state.

It is useful to compare our result for $\nu = 2/3$ with the spin transition at $\nu = 2$ which occurs when minority spin states in the lowest LL and majority spin states in the second lowest LL cross by varying the ratio of the Zeeman and Coulomb energy. The ground state at $\nu = 2$ is thus a fully polarized state or a spin singlet state. In analogous to the $\nu = 2/3$ case the transition between them is first order,²⁶⁾ and spin domain has been found in high energy states.²⁷⁾ This analogy can be expected, because the $\nu = 2$ states and the $\nu = 2/3$ states are connected in the composite fermion theory,^{3,4)} although the effective interaction between composite fermions is different from that for electrons.

Our result is consistent with a number of experimental studies. In a earlier work a downward cusp behavior of the activation gap has been observed as a function of the ratio of the Zeeman and Coulomb energy at $\nu = 2/3$.⁵⁾ Recently direct measurements of the spin configuration of the ground states have been done by using circular polarization of time-resolved luminescence, which indicate somewhat complicated situations: in addition to the fully polarized state, there is also a weak structure visible midway between the two prominent phases, corresponding to half the maximal spin polarization.⁸⁾ Motivated by this experimental finding, a number of theoretical studies have been done, concluding existence of stable half polarized states, in contrast to the present work. We claim that in experimental situations electrons strongly interact with nuclear spins.^{10–14)} We speculate that internal local magnetic fields due to nuclear spins may stabilize partially polarized states. Indeed the longitudinal resistance shows a predominantly long scale time development with the change in magnetic fields at fixed filling factors.¹²⁾ To understand the interesting behaviors of the partially polarized state in experimental situations, the effect of such external degrees should be taken into account.

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